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## ON A SOLUTION OF THE GENERAL BIQUADRATIC EQUATION.

By A. C. BURNHAM, Professor of Mathematics, University of Illinois, Urbana, Illinois.

Very often in mathematical work does one wish to write out without waste of time the value of the unknown in a given biquadratic equation. Nowhere in text-books or mathematical writings do I find the solution to a biquadratic given in such form that one by merely substituting in a formula may get the roots. I have found the formula here given convenient and I do not know that the formula or this particular method of getting the result has ever before been published.

Let the general biquadratic be

$$x^4 + a_1x^3 + a_2x^2 + a_3x + a_4 = 0,$$

and let the roots be  $a, b, c, d$ . Then follow, as is well known,

$$a + b + c + d = -a_1,$$

$$ab + ac + ad + bc + bd + cd = a_2,$$

$$abc + abd + acd + bcd = -a_3,$$

$$abcd = a_4.$$

Now let

$$\left. \begin{aligned} z_1 &= ab + cd \\ z_2 &= ac + bd \\ z_3 &= ad + bc \end{aligned} \right\} \dots\dots\dots \text{I.}$$

Then it follows that,

$$z_1 + z_2 + z_3 = a_2,$$

$$\begin{aligned} z_1z_2 + z_1z_3 + z_2z_3 &= (ab + cd)(ac + bd) + (ab + cd)(ad + bc) + (ac + bd)(ad + bc) \\ &= a^2bc + ab^2d + c^2ad + cbd^2 + \dots\dots + \dots\dots \\ &= \Sigma a^2bc = a_1a_3 - 4a_4, \end{aligned}$$

and

$$\begin{aligned} z_1z_2z_3 &= (ab + cd)(ac + bd)(ad + bc) \\ &= \Sigma a^3bcd + \Sigma a^2b^2c^2 \\ &= a_3^2 + a_1^2a_4 - 4a_2a_4. \end{aligned}$$

Then  $z_1, z_2, z_3$  are therefore the roots of the reducing cubic :

$$z^3 - a_2z^2 + (a_1a_3 - 4a_4)z - (a_3^2 + a_1^2a_4 - 4a_2a_4) = 0 \dots\dots\dots \text{II.}$$

Now from I we have

$$\begin{aligned}
 z_1^2 &= a^2 b^2 + c^2 d^2 + 2abcd \\
 &= a^2 b^2 + c^2 d^2 + 2a_4, \\
 \therefore z_1^2 - 4a_4 &= a^2 b^2 + c^2 d^2 - 2abcd = (ab - cd)^2, \\
 \therefore \sqrt{z_1^2 - 4a_4} &= ab - cd \dots\dots\dots (a), \\
 \text{but } z_1 &= ab + cd \dots\dots\dots (b).
 \end{aligned}$$

Therefore by adding (a) and (b),

$$ab = \frac{1}{2} \{ z_1 + \sqrt{z_1^2 - 4a_4} \} \dots\dots\dots (c),$$

and by subtracting (a) from (b) we have

$$cd = \frac{1}{2} (z_1 - \sqrt{z_1^2 - 4a_4}) \dots\dots\dots (d).$$

In the same manner we get

$$ac = \frac{1}{2} (z_2 + \sqrt{z_2^2 - 4a_4}) \dots\dots\dots (e),$$

$$bd = \frac{1}{2} (z_2 - \sqrt{z_2^2 - 4a_4}) \dots\dots\dots (f),$$

$$ad = \frac{1}{2} (z_3 + \sqrt{z_3^2 - 4a_4}) \dots\dots\dots (g),$$

$$bc = \frac{1}{2} (z_3 - \sqrt{z_3^2 - 4a_4}) \dots\dots\dots (h).$$

But  $ab + ac + ad = a(b + c + d)$

$$= (-a_1 - a)a, \text{ since } b + c + d = -a_1 - a$$

$$= -a^2 - a_1 a.$$

$$\text{Also } ab + ac + ad = \frac{1}{2} \{ z_1 + z_2 + z_3 + \sqrt{z_1^2 - 4a_4} + \sqrt{z_2^2 - 4a_4} + \sqrt{z_3^2 - 4a_4} \}$$

from (c), (e), and (g). Therefore,

$$a^2 + a_1 a + \frac{1}{2} \{ a_2 + \sqrt{z_1^2 - 4a_4} + \sqrt{z_2^2 - 4a_4} + \sqrt{z_3^2 - 4a_4} \} = 0,$$

which is a biquadratic equation giving the value of one root  $a$ , i. e.

$$a = \frac{-a_1 \pm \sqrt{a_2 - 2 \{ a_2 + \sqrt{z_1^2 - 4a_4} + \sqrt{z_2^2 - 4a_4} + \sqrt{z_3^2 - 4a_4} \}}}{2} \dots\dots$$

The four roots are, therefore,

$$\left. \begin{matrix} a \\ b \\ c \\ d \end{matrix} \right\} = \frac{1}{2} \left\{ -a_1 \pm \sqrt{a_1^2 - 2 \{ a_2 \pm \sqrt{z_1^2 - 4a_4} \pm \sqrt{z_2^2 - 4a_4} \pm \sqrt{z_3^2 - 4a_4} \}} \dots\dots \text{III}, \right.$$

where the sequence of signs under the main radical, as can be seen from formulæ (c) to (h), is

for  $a$ ,    +    +    +

for  $b$ ,    +    -    -

for  $c$ ,    -    +    -

for  $d$ ,    -    -    +

For the  $z_1, z_2, z_3$  in this solution III must be substituted the roots of the cubic II.

EXAMPLE. As an example take the biquadratic

$$x^4 - x^3 - 7x^2 + x + 6 = 0.$$

Here we have,

$$a_1 = -1, \quad a_3 = 1,$$

$$a_2 = -7, \quad a_4 = 6,$$

from which the cubic becomes  $z^3 + 7z^2 - 25z - 175 = 0$ , of which the roots are 5, -7, and -5. Thus the roots of the biquadratic are

$$\frac{1}{2}\{1 \pm \sqrt{1 - 2\{-7 \pm 1 \pm 5 \pm 1\}},$$

or 1, -1, -2, 3, which are seen to be correct.

Care must be exercised that the proper sign before the main radical is taken.

*Urbana, Ill., October 9, 1897.*

## EQUATION OF PAYMENTS.

By J. A. CALDERHEAD, A. B., Professor of Mathematics, Curry University, Pittsburg, Pennsylvania.

Let it be required to find the equated time of two payments,  $P$  and  $P_1$ , due at the end of  $t$  and  $t_1$  years respectively, and  $r$  being the rate of interest.

Represent the equated time by  $x$  when  $t > t_1$ .

I. BY SIMPLE INTEREST.

1st Method. The discount on  $P$  for  $(t-x)$  years must equal the interest on  $P_1$  for  $(x-t_1)$  years.

$$\frac{P(t-x)r}{1+(t-x)r} = \text{discount on } P \text{ due } (t-x) \text{ years hence.}$$

$$P_1(x-t_1)r = \text{interest on } P_1 \text{ for } (x-t_1) \text{ years.}$$

$$\therefore \frac{P(t-x)r}{1+(t-x)r} = P_1(x-t_1)r.$$